Using the proficiencies from the Australian Mathematics Curriculum to enrich mathematics teaching and assessment

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There are explicit statements in both the overall documents informing the development of the new Australian curriculum and the paper informing the development of the mathematics curriculum that affirm this emphasis on ensuring that the numeracy and practical mathematical goals are addressed. In converting these emphases into documents advising teachers, ACARA (2010) proposed that the content be arranged in three *content* strands that can be thought of as the nouns, and four *proficiency* strands that can be thought of as the verbs. The content strands, *Number and algebra*; *Measurement and geometry*; and *Statistics and probability*, represent a conventional statement of the “nouns” that are the focus of the curriculum.

More interesting, and a break from the common ways of describing such mathematical actions, are the four proficiency or process strands which were adapted from the recommendations in *Adding it up* (Kilpatrick, Swafford, & Findell, 2001). The first of these is “Understanding” (the Kilpatrick et al. term was conceptual understanding) and is described as follows:

> Students build a robust knowledge of adaptable and transferable mathematical concepts, they make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the „why‟ and the „how‟ of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

Understanding has long been a goal and teachers are familiar with its importance. Skemp (1977), for example, explained that it is not enough for students to understand how to perform various mathematical tasks; they must also appreciate why each of the ideas and relationships work the way that they do. Skemp (1971) had earlier elaborated an important idea based on the work of Piaget related to schema or mental structures. Basically, the notion is that well constructed knowledge is linked together so that when one part of a network of ideas is recalled for use at some future time, the other parts are also recalled. For example, ideally students can appreciate the meaning of the symbols, words, and relationships associated with the particular concepts and connect these different representations to each other and use them later in building new ideas.

The second of the "verbs" is *fluency* (the Kilpatrick et al. term was procedural fluency) includes:

> ... choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly used facts, and when they can manipulate expressions and equations to find solutions.

Pegg (2010) presented a clear and cogent argument for the importance of developing fluency for all students. Pegg explained that initial processing of information happens in working memory, which is of limited capacity. He focused on developing fluency in calculation as a way of reducing the load on working memory, so allowing more capacity for other mathematical actions. An example of the way this works is in mathematical language and number facts. If students do not know what is meant by terms such as parallel, right angle, index, remainder, average, then the instruction using those terms will be confusing and ineffective since so much of their working memory will be utilized trying to seek clues for the meaning of the relevant terminology. Likewise, if students can readily recall key number facts, these facts can facilitate problem solving and other actions. It can be argued that fluency is the focus of most externally set assessments, and therefore is emphasized by teachers especially in those years with external assessments to the detriment of the other proficiencies. One of the challenges facing Australian education is to find ways to give appropriate recognition in assessments to the other three proficiencies to ensure that they are appropriately emphasized by teachers.

The third of these mathematical actions is problem solving (strategic competence) which was described as:

> ... the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify their answers are reasonable.

Turner (2010) termed this "devising strategies" which he said involves

> a set of critical control processes that guide an individual to effectively recognize, formulate and solve problems. This skill is characterized as selecting or devising a plan or strategy to use mathematics to solve problems arising from a task or context, as well as guiding its implementation. (p. 59)

Problem solving has been a focus of research, curriculum and teaching for some time, and so teachers are familiar with its meaning and resources that can be used to support students learning to solve problems.

The fourth proficiency, reasoning (adaptive reasoning) includes:

> ... analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.

This action has been perhaps underemphasized in recent Australian jurisdictional curriculums. Stacey (2010) argued for the need to support the teaching of reasoning. She reported two studies. In the first of these she argued that the mathematics texts did pay some attention to proofs and reasoning, but in a way which seemed “to be to derive a rule in preparation for using it in the exercises rather than to give explanations that might be used as a thinking tool in subsequent problems” (p. 20).

Those familiar with the Kilpatrick et al. report will notice that “productive disposition” is not included in this list. The reason is that disposition was taken to refer to pedagogical approaches which were not proposed to be included in the curriculum statement.

The Kilpatrick et al. terms have been slightly simplified for ease of communication, and the proposed words are in common usage among teachers in Australian. The metaphor of verbs acting on nouns describes the explicit intention to ensure that the emphasis is on the full range of mathematical processes and not just fluency. The challenge for teachers is to find ways to incorporate a balance of these different verbs in their teaching. The conference presentation included a range of exemplars of each of these mathematical actions.

References

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