The Australian **Curriculum**

Subjects	Specialist Mathematics
Units	Unit 1, Unit 2, Unit 3 and Unit 4
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AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

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Specialist Mathematics

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Specialist Mathematics

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout Specialist Mathematics. There is also a sound logical basis to this subject, and in mastering the subject students will develop logical reasoning skills to a high level.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of Specialist Mathematics will be able to appreciate the true nature of mathematics, its beauty and its functionality.

Specialist Mathematics has been designed to be taken in conjunction with Mathematical Methods. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematical Methods and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Specialist Mathematics is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still applicable and should be inherent in students' learning of the subject. These strands are Understanding, Fluency, Problem solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem solving. In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problems, for example integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject.

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1 vectors for two-dimensional space are introduced and then in Unit 3 vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in Mathematical Methods, is applied in Vectors in Unit 3 and applications of calculus and statistics in Unit 4.

Aims

Specialist Mathematics aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately.
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve analysing univariate and bivariate data, including time series data.

Mathematical Methods focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of Specialist Mathematics

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit provides a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Unit 1 and then in Unit 3 they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn prepares students for solving simultaneous equations in three variables.

Unit 1	Unit 2	Unit 3	Unit 4
Combinatorics Vectors in the plane Geometry	Trigonometry Matrices Real and complex numbers	Complex numbers Functions and sketching graphs Vectors in three dimensions	Integration and applications of integration Rates of change and differential equations Statistical inference

Units

Unit 1 contains three topics that complement the content of Mathematical Methods. The proficiency strand, 'Reasoning', of the F–10 curriculum is continued explicitly in the topic 'Geometry' through a discussion of developing mathematical arguments. This topic also provides the opportunity to summarise and extend students' studies in Euclidean Geometry, knowledge which is of great benefit in the later study of topics such as vectors and complex numbers. The topic 'Combinatorics' provides techniques that are very useful in many areas of mathematics, including probability and algebra. The topic 'Vectors in the plane' provides new perspectives on working with two-dimensional space, and serves as an introduction to techniques which can be extended to three-dimensional space in Unit 3. These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Unit 2 contains three topics, 'Trigonometry', 'Matrices' and 'Real and complex numbers'. 'Matrices' provides new perspectives for working with two-dimensional space, 'Real and complex numbers' provides a continuation of the study of numbers. The topic 'Trigonometry' contains techniques that are used in other topics in both this unit and Units 3 and 4. All of these topics develop students' ability to construct mathematical arguments. The technique of proof by the principle of mathematical induction is introduced in this unit.

Unit 3 contains three topics, ‘Complex numbers’, ‘Vectors in three dimensions’, and ‘Functions and sketching graphs’. The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3 the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in Mathematical Methods is extended and utilised in the sketching of graphs and the solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students’ knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes,enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

Unit 4 contains three topics: ‘Integration and applications of integration’, ‘Rates of change and differential equations’ and ‘Statistical inference’. In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students’ previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Links to Foundation to Year 10

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still very much applicable and should be inherent in students’ learning of the subject. The strands of Understanding, Fluency, Problem solving and Reasoning are essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving.In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes an important role, and the ability to present the solution of any problem in a logical and clear manner is of paramount significance. The ability to transfer skills learned to solve one class of problems, such as integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject. In order to study Specialist Mathematics, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended as preparation for Specialist Mathematics:

- ACMMG273: Establish the sine, cosine and area rules for any triangle, and solve related problems
- ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies
- ACMNAP266: Investigate the concept of a polynomial, and apply the factor and remainder theorems to solve problems.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains topics that will equip students for the ever-increasing demands of the information age.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skillfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander peoples' histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Each of the senior secondary mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit Description

Unit 1 of Specialist Mathematics contains three topics – ‘Combinatorics’, ‘Vectors in the plane’ and ‘Geometry’ – that complement the content of Mathematical Methods. The proficiency strand, Reasoning, of the F–10 curriculum is continued explicitly in ‘Geometry’ through a discussion of developing mathematical arguments. While these ideas are illustrated through deductive Euclidean geometry in this topic, they recur throughout all of the topics in Specialist Mathematics. ‘Geometry’ also provides the opportunity to summarise and extend students’ studies in Euclidean Geometry. An understanding of this topic is of great benefit in the study of later topics in the course, including vectors and complex numbers.

‘Vectors in the plane’ provides new perspectives for working with two-dimensional space, and serves as an introduction to techniques that will be extended to three-dimensional space in Unit 3.

‘Combinatorics’ provides techniques that are useful in many areas of mathematics including probability and algebra. All these topics develop students’ ability to construct mathematical arguments.

These three topics considerably broaden students’ mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in combinatorics, geometry and vectors
- apply reasoning skills and solve problems in combinatorics, geometry and vectors
- communicate their arguments and strategies when solving problems
- construct proofs in a variety of contexts including algebraic and geometric
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Combinatorics

Permutations (ordered arrangements):

- solve problems involving permutations (ACMSM001)
- use the multiplication principle (ACMSM002)
- use factorial notation (ACMSM003)
- solve problems involving permutations and restrictions with or without repeated objects. (ACMSM004)

The inclusion-exclusion principle for the union of two sets and three sets:

- determine and use the formulas for finding the number of elements in the union of two and the union of three sets. (ACMSM005)

The pigeon-hole principle:

- solve problems and prove results using the pigeon-hole principle. (ACMSM006)

Combinations (unordered selections):

- solve problems involving combinations (ACMSM007)
- use the notation $\binom{n}{r}$ or ${}^nC_r$ (ACMSM008)
- derive and use simple identities associated with Pascal's triangle. (ACMSM009)

Topic 2: Vectors in the plane

Representing vectors in the plane by directed line segments:

- examine examples of vectors including displacement and velocity (ACMSM010)
- define and use the magnitude and direction of a vector (ACMSM011)
- represent a scalar multiple of a vector (ACMSM012)
- use the triangle rule to find the sum and difference of two vectors. (ACMSM013)

Algebra of vectors in the plane:

- use ordered pair notation and column vector notation to represent a vector (ACMSM014)
- define and use unit vectors and the perpendicular unit vectors *i* and *j* (ACMSM015)
- express a vector in component form using the unit vectors *i* and *j* (ACMSM016)
- examine and use addition and subtraction of vectors in component form (ACMSM017)
- define and use multiplication by a scalar of a vector in component form (ACMSM018)
- define and use scalar (dot) product (ACMSM019)
- apply the scalar product to vectors expressed in component form (ACMSM020)
- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)
- define and use projections of vectors (ACMSM022)

- solve problems involving displacement, force and velocity involving the above concepts. (ACMSM023)

Topic 3: Geometry

The nature of proof:

- use implication, converse, equivalence, negation, contrapositive (ACMSM024)
- use proof by contradiction (ACMSM025)
- use the symbols for implication ($\Rightarrow$), equivalence ($\Longleftrightarrow$), and equality ($=$) (ACMSM026)
- use the quantifiers ‘for all’ and ‘there exists’ (ACMSM027)
- use examples and counter-examples. (ACMSM028)

Circle properties and their proofs including the following theorems:

- An angle in a semicircle is a right angle (ACMSM029)
- The angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc (ACMSM030)
- Angles at the circumference of a circle subtended by the same arc are equal (ACMSM031)
- The opposite angles of a cyclic quadrilateral are supplementary (ACMSM032)
- Chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length (ACMSM033)
- The alternate segment theorem (ACMSM034)
- When two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord (ACMSM035)
- When a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant. ($AM \times BM = TM^2$) (ACMSM036)
- Suitable converses of some of the above results (ACMSM037)
- Solve problems finding unknown angles and lengths and prove further results using the results listed above. (ACMSM038)

Geometric proofs using vectors in the plane including:

- The diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
- Midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
- The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides. (ACMSM041)

Unit 2

Unit Description

Unit 2 of Specialist Mathematics contains three topics – ‘Trigonometry’, ‘Real and complex numbers’ and ‘Matrices’…

‘Trigonometry’ contains techniques that are used in other topics in both this unit and Unit 3. ‘Real and complex numbers’ provides a continuation of students’ study of numbers, and the study of complex numbers is continued in Unit 3. This topic also contains a section on proof by mathematical induction. The study of matrices is undertaken, including applications to linear transformations of the plane.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in trigonometry, real and complex numbers, and matrices
- apply reasoning skills and solve problems in trigonometry, real and complex numbers, and matrices
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Trigonometry

The basic trigonometric functions:

- find all solutions of $\mathrm{f(a(x-b))=c}$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM042)
- graph functions with rules of the form $\mathrm{y=f(a(x-b))}$ where f is one of $\sin$, $\cos$ or $\tan$. (ACMSM043)

Compound angles:

- prove and apply the angle sum, difference and double angle identities. (ACMSM044)

The reciprocal trigonometric functions, secant, cosecant and cotangent:

- define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them. (ACMSM045)

Trigonometric identities:

- prove and apply the Pythagorean identities (ACMSM046)
- prove and apply the identities for products of sines and cosines expressed as sums and differences (ACMSM047)
- convert sums $\mathrm{a}\cos\mathrm{x}+\mathrm{b}\ \sin\mathrm{x}$ to $\mathrm{R}\ \cos(\mathrm{x}\pm\alpha)$ or $\mathrm{R}\sin(\mathrm{x}\pm\alpha)$ and apply these to sketch graphs, solve equations of the form $\mathrm{a}\cos\mathrm{x}+\mathrm{b}\sin\mathrm{x}=\mathrm{c}$ and solve problems (ACMSM048)
- prove and apply other trigonometric identities such as $\cos 3\mathrm{x}=4\cos^3\mathrm{x}-3\cos\mathrm{x}$. (ACMSM049)

Applications of trigonometric functions to model periodic phenomena:

- model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model. (ACMSM050)

Topic 2: Matrices

Matrix arithmetic:

- understand the matrix definition and notation (ACMSM051)
- define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse (ACMSM052)
- calculate the determinant and inverse of 2×2 matrices and solve matrix equations of the form $\mathrm{AX=B}$, where A is a 2×2 matrix and X and B are column vectors. (ACMSM053)

Transformations in the plane:

- translations and their representation as column vectors (ACMSM054)
- define and use basic linear transformations: dilations of the form $(\mathrm{x},\mathrm{y})\longrightarrow(\lambda_1\mathrm{x},\lambda_2\mathrm{y})$, rotations about the origin and reflection in a line which passes through the origin, and the representations of these transformations by 2×2 matrices (ACMSM055)
- apply these transformations to points in the plane and geometric objects (ACMSM056)
- define and use composition of linear transformations and the corresponding matrix products (ACMSM057)
- define and use inverses of linear transformations and the relationship with the matrix inverse (ACMSM058)
- examine the relationship between the determinant and the effect of a linear transformation on area (ACMSM059)

- establish geometric results by matrix multiplications; for example, show that the combined effect of two reflections in lines through the origin is a rotation. (ACMSM060)

Topic 3: Real and complex numbers

Proofs involving numbers:

- prove simple results involving numbers. (ACMSM061)

Rational and irrational numbers:

- express rational numbers as terminating or eventually recurring decimals and vice versa (ACMSM062)
- prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$. (ACMSM063)

An introduction to proof by mathematical induction:

- understand the nature of inductive proof including the 'initial statement' and inductive step (ACMSM064)
- prove results for sums, such as $1 + 4 + 9 \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer *n* (ACMSM065)
- prove divisibility results, such as $3^{2n+4} - 2^{2n}$ is divisible by 5 for any positive integer *n*. (ACMSM066)

Complex numbers:

- define the imaginary number $\mathbf{i}$ as a root of the equation $\mathbf{x}^2 = -1$ (ACMSM067)
- use complex numbers in the form $\mathbf{a} + \mathbf{bi}$ where $\mathbf{a}$ and $\mathbf{b}$ are the real and imaginary parts (ACMSM068)
- determine and use complex conjugates (ACMSM069)
- perform complex-number arithmetic: addition, subtraction, multiplication and division. (ACMSM070)

The complex plane:

- consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates (ACMSM071)
- examine addition of complex numbers as vector addition in the complex plane (ACMSM072)
- understand and use location of complex conjugates in the complex plane. (ACMSM073)

Roots of equations:

- use the general solution of real quadratic equations (ACMSM074)
- determine complex conjugate solutions of real quadratic equations (ACMSM075)
- determine linear factors of real quadratic polynomials. (ACMSM076)

Specialist Mathematics

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts • constructs mathematical proofs in a variety of contexts, and adapts previously seen mathematical proofs • uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems • selects and applies techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs simple mathematical proofs, and adapts previously seen mathematical proofs • uses digital technologies appropriately to graph, display and organise mathematical information to solve a range of routine and non-routine problems	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • reproduces previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen simple mathematical proofs • uses digital technologies to display some mathematical information in routine problems	• demonstrates limited familiarity with simple concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical models • demonstrates limited familiarity with mathematical proofs • uses digital technologies for arithmetic calculations and to display limited mathematical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical judgments and arguments, including simple proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical and symbolic form in routine problems • communicates mathematical arguments, including previously seen proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical or symbolic form in routine problems • communicates mathematical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical information in a structured context • communicates simple mathematical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

Unit 3 of Specialist Mathematics contains three topics: ‘Vectors in three dimensions’, ‘Complex numbers’ and ‘Functions and sketching graphs’. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students’ knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in Mathematical Methods, is extended and applied in sketching graphs and solving problems involving integration.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students will:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching
- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Complex numbers

Cartesian forms:

- review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z (ACMSM077)
- review Cartesian form (ACMSM078)
- review complex arithmetic using Cartesian forms. (ACMSM079)

Complex arithmetic using polar form:

- use the modulus $|z|$of a complex number z and the argument $Arg\ (z)$ of a non-zero complex number z and prove basic identities involving modulus and argument (ACMSM080)
- convert between Cartesian and polar form (ACMSM081)
- define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these (ACMSM082)
- prove and use De Moivre's theorem for integral powers. (ACMSM083)

The complex plane (the Argand plane):

- examine and use addition of complex numbers as vector addition in the complex plane (ACMSM084)
- examine and use multiplication as a linear transformation in the complex plane (ACMSM085)
- identify subsets of the complex plane determined by relations such as$|z-3i| \leq 4$ $\frac{\pi}{4} \leq Arg\left(z\right) \leq \frac{3\pi}{4}$, $Re(z) > Im(z)$ and $|z-1| = 2|z-i|$. (ACMSM086)

Roots of complex numbers:

- determine and examine the n^{th} roots of unity and their location on the unit circle (ACMSM087)
- determine and examine the n^{th}roots of complex numbers and their location in the complex plane. (ACMSM088)

Factorisation of polynomials:

- prove and apply the factor theorem and the remainder theorem for polynomials (ACMSM089)
- consider conjugate roots for polynomials with real coefficients (ACMSM090)
- solve simple polynomial equations. (ACMSM091)

Topic 2: Functions and sketching graphs

Functions:

- determine when the composition of two functions is defined (ACMSM092)
- find the composition of two functions (ACMSM093)
- determine if a function is one-to-one (ACMSM094)
- consider inverses of one-to-one function (ACMSM095)
- examine the reflection property of the graph of a function and the graph of its inverse. (ACMSM096)
- *Sketching graphs:* (ACMSM097)

- use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y = |x|$ (ACMSM098)
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$,$y = |f(x)|$ and $y = f(|x|)$ (ACMSM099)
- sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree. (ACMSM100)

Topic 3: Vectors in three dimensions

The algebra of vectors in three dimensions:

- review the concepts of vectors from Unit 1 and extend to three dimensions including introducing the unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. (ACMSM101)
- prove geometric results in the plane and construct simple proofs in three-dimensions. (ACMSM102)

Vector and Cartesian equations:

- introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres (ACMSM103)
- use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case (ACMSM104)
- determine a vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions (ACMSM105)
- examine the position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet (ACMSM106)
- use the cross product to determine a vector normal to a given plane (ACMSM107)
- determine vector and Cartesian equations of a plane and of regions in a plane. (ACMSM108)

Systems of linear equations:

- recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations (ACMSM109)
- examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables. (ACMSM110)

Vector calculus:

- consider position of vectors as a function of time (ACMSM111)
- derive the Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas (ACMSM112)
- differentiate and integrate a vector function with respect to time (ACMSM113)
- determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration (ACMSM114)
- apply vector calculus to motion in a plane including projectile and circular motion. (ACMSM115)

Unit 4

Unit Description

Unit 4 of Specialist Mathematics contains three topics: 'Integration and applications of integration', 'Rates of change and differential equations' and 'Statistical inference'.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout Specialist Mathematics.

In this unit all of the students' previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference
- apply reasoning skills and solve problems in applications of calculus and statistical inference
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Integration and applications of integration

Integration techniques:

- integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}\left(1 - \cos 2x\right)$, $\cos^2 x = \frac{1}{2}\left(1 + \cos 2x\right)$ and $1 + \tan^2 x = \sec^2 x$ (ACMSM116)
- use substitution *u* = *g*(*x*) to integrate expressions of the form $f(g(x))g'(x)$ (ACMSM117)
- establish and use the formula $\int \frac{1}{x}\, dx = \ln |x| + c$, for $x \neq 0$ (ACMSM118)
- find and use the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM119)
- find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM120)
- integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ (ACMSM121)
- use partial fractions where necessary for integration in simple cases (ACMSM122)
- integrate by parts. (ACMSM123)

Applications of integral calculus:

- calculate areas between curves determined by functions (ACMSM124)
- determine volumes of solids of revolution about either axis (ACMSM125)
- use numerical integration using technology (ACMSM126)
- use and apply the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and use the exponential random variables and associated probabilities and quantiles to model data and solve practical problems. (ACMSM127)

Topic 2: Rates of change and differential equations

- use implicit differentiation to determine the gradient of curves whose equations are given in implicit form (ACMSM128)
- Related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (ACMSM129)
- solve simple first-order differential equations of the form $\frac{dy}{dx} = f\left(x\right)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables (ACMSM130)
- examine slope (direction or gradient) fields of a first order differential equation (ACMSM131)
- formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved. (ACMSM132)

Modelling motion:

- examine momentum, force, resultant force, action and reaction (ACMSM133)
- consider constant and non-constant force (ACMSM134)
- understand motion of a body under concurrent forces (ACMSM135)
- consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$ for acceleration. (ACMSM136)

Topic 3: Statistical inference

Sample means:

- examine the concept of the sample mean X as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ (ACMSM137)
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large (ACMSM138)
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where s is the sample standard deviation. (ACMSM139)

Confidence intervals for means:

- understand the concept of an interval estimate for a parameter associated with a random variable (ACMSM140)
- examine the approximate confidence interval $\left(\overline{\mathrm{X}} - \frac{\mathrm{zs}}{\sqrt{n}},\ \overline{\mathrm{X}} + \frac{\mathrm{zs}}{\sqrt{n}}\right)$, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution (ACMSM141)
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ (ACMSM142)
- use $\overline{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ (ACMSM143)
- collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality. (ACMSM144)

Units 3 and 4 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of concepts of functions, calculus, vectors and statistics in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts • constructs mathematical proofs in a variety of contexts using a range of techniques • uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics in routine and non-routine problems • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs mathematical proofs in a variety of contexts and adapts previously seen mathematical proofs • uses digital technologies appropriately to graph, display and organise mathematical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • constructs simple mathematical proofs and adapts previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen mathematical proofs • uses digital technologies to display some mathematical information in routine problems	• demonstrates limited familiarity with simple concepts of functions, calculus, vectors and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical models • reproduces previously seen simple mathematical proofs • uses digital technologies for arithmetic calculations and to display limited mathematical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments, including simple proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates mathematical and statistical arguments, including previously seen proofs, using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical and statistical information in a structured context • communicates simple mathematical and statistical information using appropriate language • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Specialist Mathematics Glossary

(Multiplicative) identity matrix

A (multiplicative)identity matrix is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter I.

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with a subscript: I_n

Addition of matrices (See Matrix)

If A and B are matrices with the same dimensions and the entries of A are a_{ij} and the entries of B are b_{ij} then the entries of $A + B$ are $a_{ij} + b_{ij}$

For example if

$$\mathrm{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix} \text{ then}$$

$$\mathrm{A} + \mathrm{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$$

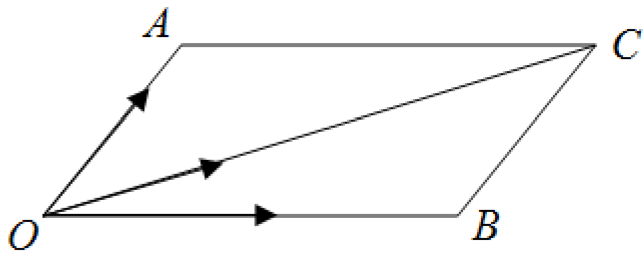
Addition of vectors

In component form if a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a + b = (a_1 + b_1)i + (a_2 + b_2)j + (a_3+ b_3)k

Addition of vectors (see Vector for definition and notation)

Given vectors a and b let $\vec{OA}$ and $\vec{OB}$ be directed line segments that represent a and b. They have the same initial point *O*. The sum of $\vec{OA}$ and $\vec{OB}$ is the directed line segment $\vec{OC}$where *C* is a point such that *OACB* is a parallelogram. This is known as the parallelogram rule.



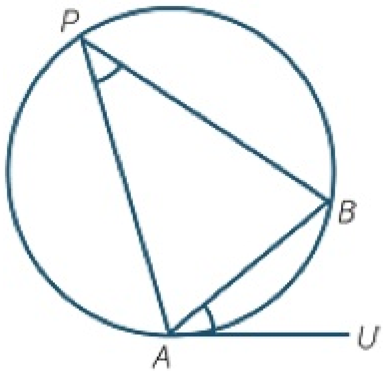
If a = (a_1, a_2) and b = (b_1, b_2) then a + b = (a_1 + b_1, a_2 + b_2)

In component form if a = a_1i + a_2j and b = b_1i + b_2j then

a + b = (a_1 + b_1)i + (a_2+ b_2)j

Alternate segment

The word 'alternate' means 'other'. The chord *AB* divides the circle into two segments and *AU* is tangent to the circle. Angle *APB* 'lies in' the segment on the other side of chord *AB* from angle *BAU.* We say that it is in the alternate segment.



Angle sum and difference identities

sin (*A* + *B*) = sin *A* cos *B* + sin *B* cos *A*

sin(*A* – *B*) = sin *A* cos *B* – sin *B* cos *A*

cos(A + *B*) = cos *A* cos *B* – sin *A* sin *B*

cos(A – *B*) = cos *A* cos *B* + sin *A* sin *B*

Argument (abbreviated arg)

If a complex number is represented by a point *P* in the complex plane then the argument of *z*, denoted arg *z*, is the angle θ that *OP* makes with the positive real axis O_x, with the angle measured anticlockwise from O_x. The principalvalue of the argument is the one in the interval (–π, π].

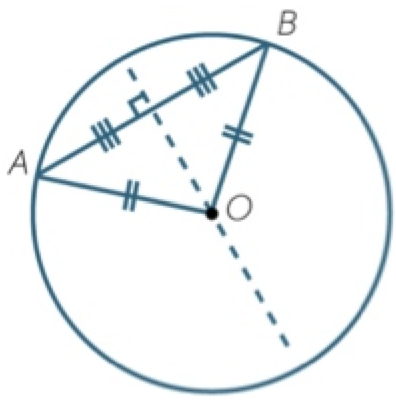
Arranging n objects in an ordered list

The number of ways to arrange n different objects in an ordered list is given by

$n(n - 1)(n - 2) \times \ldots \times 3 \times 2 \times 1 = n!$

Circle Theorems

Result 1



Let *AB* be a chord of a circle with centre *O*.

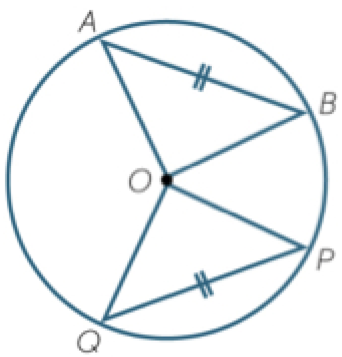
The following three lines coincide:

The bisector of the angle ∠*AOB* subtended at the centre by the chord.

The line segment (interval) joining *O* and the midpoint of the chord *AB*.

The perpendicular bisector of the chord *AB*.

Result 2

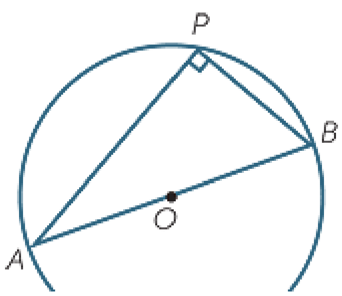


Equal chords of a circle subtend equal angles at the centre.

In the diagram shown ∠*AOB* = ∠*POQ*.

Result 3

An angle in a semicircle is a right angle.



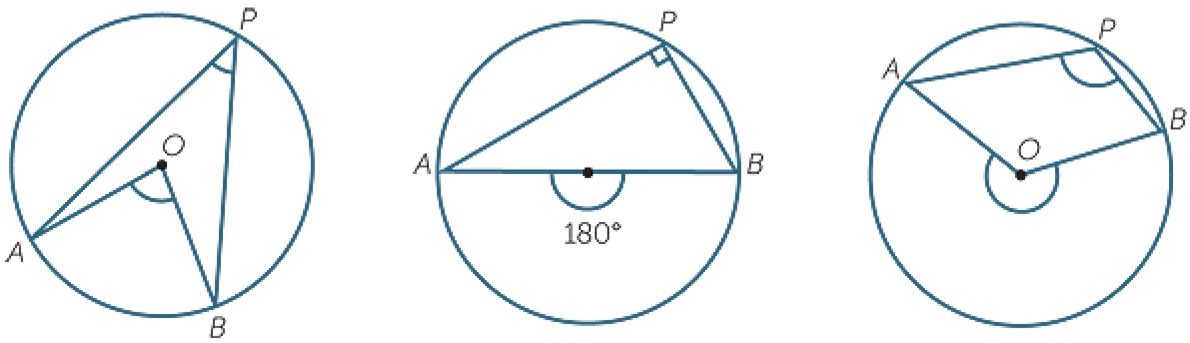
Let *AOB* be a diameter of a circle with centre *O*, and let *P* be any other point on the circle. The angle ∠*APB* subtended at *P* by the diameter *AB* is called an angle in a semicircle.

Converse

The circle whose diameter is the hypotenuse of a right-angled triangle passes through all three vertices of the triangle.

Result 4

An angle at the circumference of a circle is half the angle subtended at the centre by the same arc. In the diagram shown ∠*AOB* = 2∠*APB*

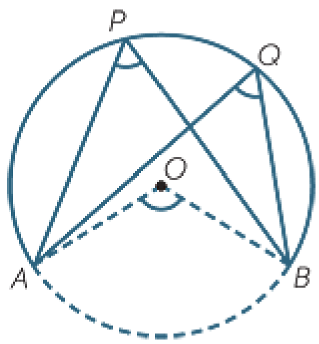


The arc *AB*subtends the angle ∠*AOB* at the centre. The arc also subtends the angle ∠*APB*, called an angle at the circumference subtended by the arc *AB*.

Result 5

Two angles at the circumference subtended by the same arc are equal.

∠APB = ∠AQB



In the diagram, the two angles ∠*APB* and ∠*AQB* are subtended by the same arc *AB*.

Result 6

The opposite angles of a cyclic quadrilateral are supplementary.

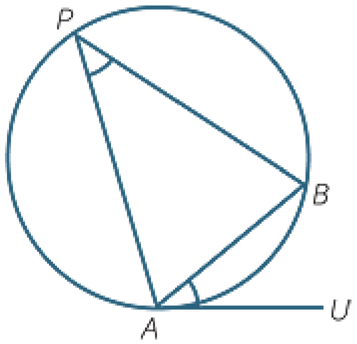
Converse

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

Result 7

Alternate segment theorem

An angle between a chord and a tangent is equal to any angle in the alternate segment.



In the diagram $\angle BAU = \angle APB$.

Combinations (Selections)

The number of selections of n objects taken r at a time (that is, the number of ways of selecting r objects out of n) is denoted by

${}^nC_r = \binom{n}{r}$ and is equal to $\frac{n!}{r!(n-r)!}$

Complex arithmetic

If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$

$z_1 + z_2 = x_1 + x_2 + (y_1 + y_2)\, i$

$z_1 - z_2 = x_1 - x_2 + (y_1 - y_2)\, i$

$z_1 \times z_2 = x_1 x_2 - y_1 y_2 + (x_1 y_2 + x_2 y_1) i$

$z_1 \times (0 + 0i) = 0$ Note: $0 + 0i$ is usually written as 0

$z_1 \times (1 + 0i) = z_1$ Note: $1 + 0i$ is usually written as 1

Complex conjugate

For any complex number $z = x + iy$, its conjugate is $\overline{z} = x - iy$. The following properties hold

$\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$

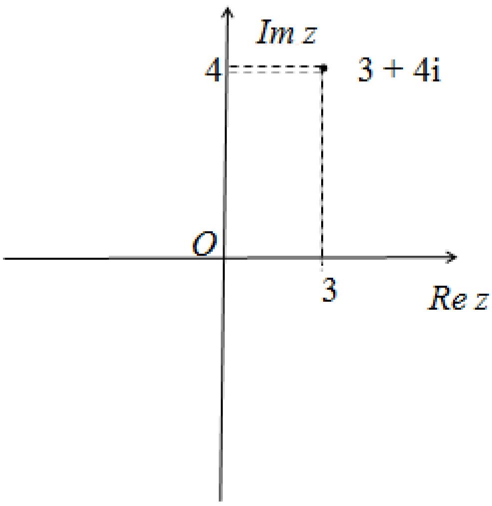
$\overline{z_1/z_2} = \overline{z_1}\ /\ \overline{z_2}$

$\overline{z} = [z]^2$

$z + \overline{z}$ is real

Complex plane (Argand plane)

The complex plane is a geometric representation of the complex numbers established by the real axis and the orthogonal imaginary axis. The complex plane is sometimes called the Argand plane.



Continuous random variable

A random variable X is called continuous if its set of possible values consists of intervals, and the chance that it takes any point value is zero (in symbols, if $P(X = x) = 0$ for every real number x). A random variable is continuous if and only if its cumulative probability distribution function can be expressed as an integral of a function.

Contradiction-Proof by

Assume the opposite (negation) of what you are trying to prove. Then proceed through a logical chain of argument till you reach a demonstrably false conclusion. Since all the reasoning is correct and a false conclusion has been reached the only thing that could be wrong is the initial assumption. Therefore the original statement is true.

For example: the result $\sqrt{2}$ is irrational can be proved in this way by first assuming $\sqrt{2}$ is rational.

The following are examples of results that are often proved by contradiction:

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

If an interval (line segment) subtends equal angles at two points on the same side of the interval (line segment), then the two points and the endpoints of the interval are concyclic.

Implication: if P then Q Symbol: $P \Rightarrow Q$

Examples:

If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other.

If $x = 2$ then $x^2 = 4$.

If an animal is a kangaroo then it is a marsupial.

If a quadrilateral is cyclic then the opposite angles are supplementary.

Converse of a statement

The converse of the statement 'If P then Q' is 'If Q then P' Symbolically the converse of $P \Rightarrow Q$ is: $Q \Rightarrow P$ or $P \Leftarrow Q$

The converse of a true statement need not be true.

Examples:

Statement: If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other.

Converse statement: If the diagonals of a quadrilateral are of equal length and bisect each other then the quadrilateral is a rectangle. (In this case the converse is true.)

Statement: If $x = 2$ then $x^2 = 4$.

Converse statement: If $x^2 = 4$ then $x = 2$. (In this case the converse is false.)

Statement: If an animal is a kangaroo then it is a marsupial.

Converse statement: If an animal is a marsupial then it is a kangaroo. (In this case the converse is false.)

Statement: If a quadrilateral is cyclic then the opposite angles are supplementary.

Converse statement: If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic. (In this case the converse is true.)

Contrapositive

The contrapositive of the statement ‘If P then Q’ is ‘If not Q then not P’. The contrapositive of a true statement is also true. (not Q is the negation of the statement Q)

Examples:

Statement: A rectangle is a quadrilateral that has diagonals of equal length and the diagonals bisect each other.

Contrapositive: If the diagonals of a quadrilateral are not of equal length or do not bisect each other then the quadrilateral is not a rectangle.

Statement: If $x = 2$ then $x^2 = 4$.

Contrapositive: If $x^2 \neq 4$ then $x \neq 2$.

Statement: A kangaroo is a marsupial.

Contrapositive: If an animal is not a marsupial then it is not a kangaroo.

Statement: The opposite angles of a cyclic quadrilateral are supplementary

Contrapositive: If the opposite angles of quadrilateral are not supplementary then the quadrilateral is not cyclic.

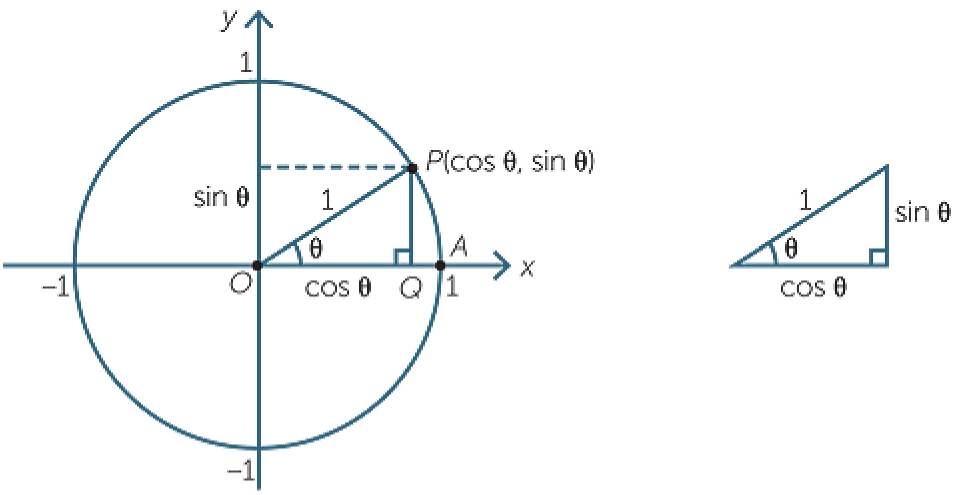
Cosine and sine functions

Since each angle θ measured anticlockwise from the positive *x*-axis determines a point P on the unit circle, we will define

the cosine of θ to be the x-coordinate of the point *P*

the sine of θ to be the y-coordinate of the point *P*

the tangent of θ is the gradient of the line segment *OP*



Counterexample

A Counterexample is an example that demonstrates that a statement is not true.

Examples:

Statement: If $x^2 = 4$ then $x = 2$.

Counterexample: $x = -2$ provides a counterexample.

Statement: If the diagonals of a quadrilateral intersect at right angles then the quadrilateral is a rhombus.

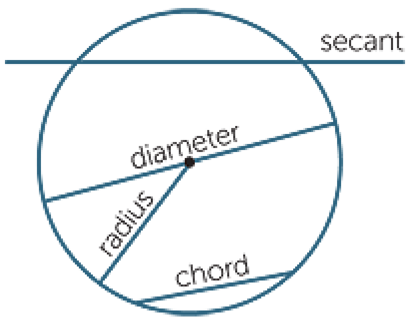
Counterexample: A kite with the diagonals not bisecting each other is not a rhombus. Such a kite provides a counterexample to the statement. The diagonals of a kite do intersect at right angles.

Statement: Every convex quadrilateral is a cyclic quadrilateral.

Counterexample: A parallelogram that is not a rectangle is convex, but not cyclic.

Cyclic quadrilateral

A cyclic quadrilateral is a quadrilateral whose vertices all lie on a circle.



Lines and line segments associated with circles

Any line segment joining a point on the circle to the centre is called a radius. By the definition of a circle, any two radii have the same length called the radius of the circle. Notice that the word ‘radius’ is used to refer both to these intervals and to the common length of these intervals.

An interval joining two points on the circle is called a chord.

A chord that passes through the centre is called a diameter. Since a diameter consists of two radii joined at their endpoints, every diameter has length equal to twice the radius. The word ‘diameter’ is use to refer both to these intervals and to their common length.

A line that cuts a circle at two distinct points is called a secant. Thus a chord is the interval that the circle cuts off a secant, and a diameter is the interval cut off by a secant passing through the centre of a circle.

De Moivre’s Theorem

For all integers n, $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$.

Determinant of a 2 × 2 matrix (See Matrix)

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant of A denoted as det A = ad – bc.

If det $A \neq 0$,

the matrix A has an inverse.

the simultaneous linear equations $ax + by = e$ and $cx + dy = f$ have a unique solution.

The linear transformation of the plane, defined by A maps the unit square

$O(0, 0)$, $B(0,1)$, $C(1, 1)$, $D(1, 0)$ to a parallelogram $OB'C'D'$ of area |det A |.

The sign of the determinant determines the orientation of the image of a figure under the transformation defined by the matrix.

Dimension (or size) (See Matrix)

Two matrices are said to have the same dimensions (or size) if they have the same number of rows and columns.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \; and \; \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same dimensions. They are both 2 × 3 matrices.

An $m \times n$ matrix has m rows and n columns.

Double angle formula

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$$\tan 2A = \frac{2\tan A}{1 - tan^2 A}$$

Entries (Elements) of a matrix

The symbol a_{ij} represents the (i, j) entry which occurs in the i^{th} row and the j^{th} column.

For example a general 3 × 2 matrix is:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

and a32 is the entry in the third row and the second column.

Equivalent statements

Statements P and Q are equivalent if P ⇒ Q and Q ⇒ P. The symbol ⇔ is used. It is also written as P if and only if Q or P iff Q.

Examples:

A quadrilateral is a rectangle if and only if the diagonals of the quadrilateral are of equal length and bisect each other.

A quadrilateral is cyclic if and only if opposite angles are supplementary.

Negation

If P is a statement then the statement ‘not P’, denoted by ¬P is the negation of P. If P is the statement ‘It is snowing.’ then ¬P is the statement ‘It is not snowing.’.

Imaginary part of a complex number

A complex number *z* may be written as *x* + *yi*, where *x* and *y* are real, and then *y* is the imaginary part of *z*. It is denoted by *Im* (*z*).

Implicit differentiation

When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. Implicit differentiation consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example,

if $x^2 + xy^3 - 2x + 3y = 0,$

then $2x + x\left(3y^2\right)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0,$

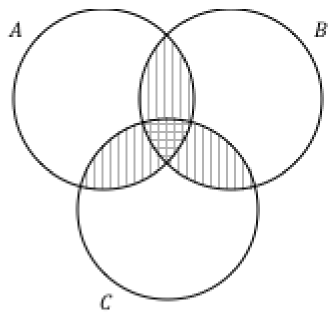
and so $\frac{dy}{dx} = \frac{2-2x-y^3}{3xy^2+3}$

Inclusion – exclusion principle

Suppose *A* and *B* are subsets of a finite set *X* then

|*A* ∪ *B* | = | *A* | + | *B* | – | *A* ∩ *B*|

Suppose *A*, *B* and *C* are subsets of a finite set *X* then



|*A* ∪ *B* ∪ *C*| = | *A* | + | *B* | + | *C* | – | *A* ∩ *B*| – | *A* ∩ *C*| – | *B* ∩ *C*| + | *A* ∩ *B* ∩ *C*|

This result can be generalised to 4 or more sets.

Independent and identically distributed observations

For independent observations, the value of any one observation has no effect on the chance of values for all the other observations. For identically distributed observations, the chances of the possible values of each observation are governed by the same probability distribution.

Integers

The integers are the numbers $\cdots, -3, \; -2, \; -1, \; 0, \; 1, \; 2, \; 3, \cdots$.

Modulus (Absolute value) of a complex number

If *z* is a complex number and *z* = *x* + *iy* then the modulus of *z* is the distance of *z* from the origin in the Argand plane. The modulus of *z* denoted by |*z* | = $\sqrt{x^2 + y^2}$.

Leading diagonal

The leading diagonal of a square matrix is the diagonal which runs from the top left corner to the bottom right corner.

Linear momentum

The linear momentum p of a particle is the vector quantity p = *m*v where *m* is the mass and v is the velocity.

Linear transformation defined by a 2 × 2 matrix

The matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax & + & by \\ cx & & dy \end{bmatrix}$$

defines a transformation *T*(*x*, *y*) = (*ax* + *by*, *cx* + *dy*).

Linear transformations in 2 dimensions

A linear transformation in the plane is a mapping of the form

T(*x*, *y*) = (*ax* + *by*, *cx* + *dy*).

A transformation *T* is linear if and only if

$T(\alpha(x_1, y_1) + \beta(x_2, y_2)) = \alpha T((x_1, y_1)) + \beta T(x_2, y_2))$.

Linear transformations include:

rotations around the origin

reflections in lines through the origin

dilations.

Translations are not linear transformations.

Logistic equation

The logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics.

One form of this differential equation is:

$\frac{dy}{dt}$ = ay - by^2 (where $a > 0$ and $b > 0$)

The general solution of this is

$y = \frac{a}{b+C_e^{-at}}$ where C is an arbitrary constant.

Magnitude of a vector (see Vector for definition and notation)

The magnitude of a vector a is the length of any directed line segment that represents a. It is denoted by |a|.

Matrix (matrices)

A matrix is a rectangular array of elements or entries displayed in rows and columns.

For example,

$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices.

Matrix A is said to be a 3 × 2 matrix (three rows and two columns) while B is said to be a 2 × 3 matrix (two rows and three columns).

A square matrix has the same number of rows and columns.

A column matrix (or vector) has only one column.

A row matrix (or vector) has only one row.

Matrix algebra of 2 × 2 matrices

If A, B and C are 2 × 2 matrices, I the 2 × 2 (multiplicative) identity matrix and O the 2 × 2 zero matrix then:

$A + B = B + A$ (commutative law for addition)

$(A + B) + C = A + (B + C)$ (associative law for addition)

$A + O = A$ (additive identity)

$A + (-A) = O$ (additive inverse)

$(AB)C = A(BC)$ (associative law for multiplication)

$AI = A = IA$ (multiplicative identity)

$A(B + C) = AB + AC$ (left distributive law)

$(B + C)A = BA + CA$ (right distributive law)

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix. The product AB of two matrices A and B with dimensions$m \times n$ and $p \times q$ is defined if $n = p$. If it is defined, the product AB is an $m \times q$ matrix and it is computed as shown in the following example.

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 6 & 10 \\ 11 & 3 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 94 & 34 \\ 151 & 63 \end{bmatrix}$$

The entries are computed as shown $1 \times 6 + 8 \times 11 + 0 \times 12 = 94$

$1 \times 10 + 8 \times 3 + 0 \times 4 = 34$

$2 \times 6 + 5 \times 11 + 7 \times 12 = 151$

$2 \times 10 + 5 \times 3 + 7 \times 4 = 63$

The entry in row i and column j of the product AB is computed by 'multiplying' row i of A by column j of B as shown.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & ba_{23} \end{bmatrix}$ then

$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{12}b_{12} + a_{22}b_{22} & a_{12}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$

Modulus of a complex number

(Modulus of a complex number Unit 2)

Multiplication by a scalar

Let a be a non-zero vector and k a positive real number (scalar) then the scalar multiple of a by k is the vector ka which has magnitude $|k||$a$|$ and the same direction as a. If k is a negative real number, then k a has magnitude $|k|$ $|$a$|$ and but is directed in the opposite direction to a. (see negative of a vector)

Some properties of scalar multiplication are:

k (a + b) = k a + k b

h(k (a)) = (hk)a

1a = a

Multiplication principle

Suppose a choice is to be made in two stages. If there are a choices for the first stage and b choices for the second stage, no matter what choice has been made at the first stage, then there are ab choices altogether. If the choice is to be made in n stages and if for each i, there are a_i choices for the i^{th} stage then there are $a_1a_2 \ldots a_n$ choices altogether.

Multiplicative inverse of a square matrix

The inverse of a square matrix A is written as A^{-1} and has the property that

$AA^{-1}= A^{-1}A= I$

Not all square matrices have an inverse. A matrix that has an inverse is said to be invertible.

multiplicative inverse of a 2 × 2 matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\boldsymbol{is}\ \boldsymbol{A}^{-1}\ \frac{1}{det\boldsymbol{A}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when det A ≠ 0.

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product:

$$10 \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix A with entries a_{ij} the entries of kA are ka_{ij}.

Negative of a vector (see Vector for definition and notation)

Given a vector a, let $\vec{AB}$ be a directed line segment representing a. The negative of a, denoted by –a, is the vector represented by $\vec{BA}$. The following are properties of vectors involving negatives:

a + (–a) = (–a) + a = 0

–(–a) = a

Pascal's triangle

Pascal's triangle is an arrangement of numbers. In general the n^{th} row consists of the binomial Coefficients

${}^{n}C_{r}$ or $\binom{n}{r}$ with the r = 0, 1,…, n

				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

In Pascal's triangle any term is the sum of the two terms 'above' it.

For example 10 = 4 + 6.

Identities include:

The recurrence relation, ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$

${}^{n}C_{k} = \frac{n}{k}\,{}^{n-1}C_{k-1}$

Permutations

A permutation of n objects is an arrangement or rearrangement of n objects (order is important). The number of arrangements of n objects is $n!$ The number of permutations of n objects taken r at a time is denoted ${}^{n}P_{r}$ and is equal to

$n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!}$

Pigeon-hole principle

If there are n pigeon holes and $n + 1$ pigeons to go into them, then at least one pigeon hole must get 2 or more pigeons.

Polar form of a complex number

For a complex number z, let θ = arg z. Then $z = r(\cos\theta + i\sin\theta)$ is the polar form of z.

Precision

Precision is a measure of how close an estimator is expected to be to the true value of the parameter it purports to estimate.

Prime numbers

A prime number is a positive integer greater than 1 that has no positive integer factor other 1 and itself. The first few prime numbers are $2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23, \cdots.$

Principle of mathematical induction

Let there be associated with each positive integer *n*, a proposition *P*(*n*).

If

P(1) is true, and

for all *k*, *P*(*k*) is true implies *P*(*k* + 1) is true,

then *P*(*n*) is true for all positive integers n.

Probability density function

The probability density function (pdf) of a continuous random variable is the function that when integrated over an interval gives the probability that the continuous random variable having that pdf, lies in that interval. The probability density function is therefore the derivative of the (cumulative probability) distribution function.

Products as sums and differences

$$\cos A \cos B = \frac{1}{2}\left(\cos\left(A - B\right) + \cos\left(A + B\right)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos\left(A - B\right) - \cos\left(A + B\right)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin\left(A + B\right) + \sin\left(A - B\right)\right)$$

$$\cos A \sin B = \frac{1}{2}\left(\sin\left(A + B\right) - \sin\left(A - B\right)\right)$$

Properties of vector addition:

a + b = b + a (commutative law)

(a + b) + c = a + (b + c) (associative law)

a + 0 = 0 + a = a

a + (–a) = 0

Pythagorean identities

$\cos^2 A + \sin^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$\cot^2 A + 1 = \mathrm{cosec}^2\, A$

Quantifiers

For all (For each)

Symbol ∀

For all real numbers x, $x^2 \geq 0$. (∀ real numbers x, $x^2 \geq 0$.)

For all triangles the sum of the interior angles is 180° .(∀ triangles the sum of the interior angles is 180°.)

For each diameter of a given circle each angle subtended at the circumference by that diameter is a right angle.

There exists

Symbol ∃

There exists a real number that is not positive (∃ a real number that is not positive.)

There exists a prime number that is not odd. (∃ a prime number that is not odd.)

There exists a natural number that is less than 6 and greater than 3.

There exists an isosceles triangle that is not equilateral.

The quantifiers can be used together.

For example: $\forall\ x \geq 0,\ \exists\ y \geq 0$ such that $y^2 = x$.

Random sample

A random sample is a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation. The most common situation in which the term "random sample" is used refers to a set of independent and identically distributed observations.

Sample mean

The arithmetic average of the sample values

Rational function

A rational function is a function such that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials. Usually $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of f is usually taken to be

$R \backslash \{x : h(x) = 0\}$.

Rational numbers

A real number is rational if it can be expressed as a quotient of two integers. Otherwise it is called irrational.

Irrational numbers can be approximated as closely as desired by rational numbers, and most electronic calculators use a rational approximation when performing calculations involving an irrational number.

Real numbers

The numbers generally used in mathematics, in scientific work and in everyday life are the real numbers. They can be pictured as points on a number line, with the integers evenly spaced along the line, and a real number a to the right of a real number b if $a > b$.

A real number is either rational or irrational. The set of real numbers consists of the set of all rational and irrational numbers.

Every real number has a decimal expansion. Rational numbers are the ones whose decimal expansions are either terminating or eventually recurring.

Real part of a complex number

A complex number z may be written as $x + yi$, where x and y are real, and then x is the real part of z. It is denoted by $Re\,(z)$.

Reciprocal trigonometric functions

$\mathbf{sec}\,\boldsymbol{A} = \frac{1}{cos\ A}$, $cos\ A \neq 0$

$\text{cosec}\ A = \frac{1}{sin\ A}$, $sin\ A \neq 0$

$\cot A = \frac{\cos A}{sin\ A}$, $sin\ A \neq 0$

Root of unity (nth root of unity)

A complex number z such that $z^n = 1$

The n^{th} roots of unity are:

$\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$ where $k = 0, 1, 2, \ldots, n - 1$.

The points in the complex plane representing roots of unity lie on the unit circle.

The cube roots of unity are

$$z_1 = 1,\ z_2 = \frac{1}{2}\left(-1 + i\sqrt{3}\right),\quad z_3 = \frac{1}{2}\left(-1 - i\sqrt{3}\right)$$

Note $z_3 = \overline{z_2}$ and $z_3 = \frac{1}{z_2}$ and $z_2 z_3 = 1$

Scalar product

If a = (a_1, a_2, a_3) and b = (b_1, b_2, b_3) then the scalar product a.b is the real number

$a_1b_1 + a_2b_2 + a_3b_3$.

When expressed in i, j, k notation, a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a.b = $a_1b_1 + a_2b_2 + a_3b_3$

Scalar product (see Vector for definition and notation)

a = (a_1, a_2) and b = (b_1, b_2) then the scalar product a.b is the real number

a_1 b_1 + a_2 b_2. The geometrical interpretation of this number is a.b =|a||b|cos (θ) where θ is the angle 'between' a and b

When expressed in i, j, notation, if a = a_1i + a_2j and b = b_1i + b_2j then

a.b = a_1 b_1 + a_2 b_2

Note |a| = $\sqrt{a.a}$

Separation of variables

Differential equations of the form $\frac{dy}{dx} = g(x)h\left(y\right)$ can be rearranged as long as $h(y) \neq 0$ to obtain

$$\frac{1}{h\left(y\right)} \frac{dy}{dx} = g\left(x\right)$$

Singular matrix

A matrix is singular if det A = 0. A singular matrix does not have a multiplicative inverse.

Slope field

Slope field (direction or gradient field) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment of the corresponding slope

Subtraction of vectors (see Vector for definition and notation)

a – b = a + (–b)

Unit vector (see Vector for definition and notation)

A unit vector is a vector with magnitude 1. Given a vector a the unit vector in the same direction as a is $\frac{1}{|a|}$

a. This vector is often denoted as $\hat{a}$.

Vector equation of a plane

Let a be a position vector of a point A in the plane, and n a normal vector to the plane . Then the plane consists of all points P whose position vector p satisfies

(p – a).n = 0. This equation may also be written as p.n =a.n, a constant.

(If the the normal vector n is the vector (l, m, n) in ordered triple notation and the scalar product

a.n = k, this gives the Cartesian equation $lx + my + nz = k$ for the plane)

Vector equation of a straight line

Let a be the position vector of a point on a line *l*, and u any vector with direction along the line. The line consists of all points *P* whose position vector p is given by

p = a + *t*u for some real number *t*.

(Given the position vectors of two points on the plane a and b the equation can be written as

p = a + *t*(b – a) for some real number *t*.)

Vector function

In this course a vector function is one that depends on a single real number parameter t, often representing time, producing a vector *r*(*t*) as the result. In terms of the standard unit vectors i, j, k of three dimensional space, the vector-valued functions of this specific type are given by expressions such as

r(*t*) = *f*(*t*)i + *g*(*t*)j + *h*(*t*)k

where *f*, *g* and *h* are real valued functions giving coordinates.

Vector product (Cross product)

When expressed in i, j, k notation, a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a × b = ($a_2b_3 - a_3b_2$)i + ($a_3b_1 - a_1b_3$)j + ($a_1b_2 - a_2b_1$)k

The cross product has the following geometric interpretation. Let a and b be two non- parallel vectors then |a × b| is the area of the parallelogram defined by a and b and

a × b is normal to this parallelogram.

(The cross product of two parallel vectors is the zero vector.)

The inverse sine function, $y = \sin^{-1} x$

If the domain for the sine function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a one to one function is formed and so an inverse function exists.

The inverse of this restricted sine function is denoted by $\sin^{-1}$ and is defined by:

$\sin^{-1}: [-1, 1] \rightarrow R$, $\sin^{-1} x = y$ where $\sin y = x$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}$ is also denoted by arcsin.

The inverse cosine function, $y = \cos^{-1} x$

If the domain of the cosine function is restricted to [0, π] a one to one function is formed and so the inverse function exists.

$\cos^{-1} x$, the inverse function of this restricted cosine function, is defined as follows:

$\cos^{-1}: [-1, 1] \rightarrow R$, $\cos^{-1} x = y$ where $\cos y = x$, $y \in [0, \pi]$

$\cos^{-1}$ is also denoted by arccos.

The inverse tangent function, $y = \tan^{-1} x$

If the domain of the tangent function is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ a one to one function is formed and so the inverse function exists.

$\mathrm{Tan}^{-1}: R \rightarrow R$, $\tan^{-1} x = y$ where $\tan y = x$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Tan^{-1} is also denoted by arctan.

Vector projection (see Vector for definition and notation)

Let a and b be two vectors and write θ for the angle between them. The projection of a vector a on a vector b is the vector

|a| cos θ $\hat{b}$ where $\hat{b}$ is the unit vector in the direction of b.

The projection of a vector a on a vector b is (a. $\hat{b}$) $\hat{b}$ where $\hat{b}$ is the unit vector in the direction of b. This projection is also given by the formula $\frac{a.b}{a.b}\,b$.

In Physics the name vector is used to describe a physical quantity like velocity or force that has a magnitude and direction.

A vector is an entity a which has a given length (magnitude) and a given direction. If $\vec{AB}$ is a directed line segment with this length and direction, then we say that $\vec{AB}$represents a.

If $\vec{AB}$ and $\vec{CD}$ represent the same vector, they are parallel and have the same length.

The zero vector is the vector with length zero.

In two dimensions, every vector can be represented by a directed line segment which begins at the origin. For example, the vector $\vec{BC}$ from B(1,2) to C(5,7) can be represented by the directed line segment $\vec{OA}$, where A is the point (4,5). The ordered pair notation for a vector uses the co-ordinates of the end point of this directed line segment beginning at the origin to denote the vector, so

$\vec{BC}$ = (4,5) in ordered pair notation. The same vector can be represented in column vector notation as .

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Vectors in three-dimensions

(See Vectors in Unit 2)

Whole numbers

A whole number is a non–negative integer, that is, one of the numbers $0,\ 1,\ 2,\ 3, \cdots$,

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ And $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

There is a zero matrix for each size of matrix. When clarity is needed we write for the *n* × *m* zero matrix. $\mathbf{O}_{n \times m}$

Achievement Standards Glossary

Glossary

Abstract

Abstract scenario: a scenario for which there is no concrete referent provided.

Account

Account for: provide reasons for (something).

Give an account of: report or describe an event or experience.

Taking into account: considering other information or aspects.

Analyse

Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences.

Apply

Use, utilise or employ in a particular situation.

Assess

Determine the value, significance or extent of (something).

Coherent

Orderly, logical, and internally consistent relation of parts.

Communicates

Conveys knowledge and/or understandings to others.

Compare

Estimate, measure or note how things are similar or dissimilar.

Complex

Consisting of multiple interconnected parts or factors.

Considered

Formed after careful thought.

Critically analyse

Examine the component parts of an issue or information, for example the premise of an argument and its plausibility, illogical reasoning or faulty conclusions

Critically evaluate

Evaluation of an issue or information that includes considering important factors and available evidence in making critical judgement that can be justified.

Deduce

Arrive at a conclusion by reasoning.

Demonstrate

Give a practical exhibition as an explanation.

Describe

Give an account of characteristics or features.

Design

Plan and evaluate the construction of a product or process.

Develop

In history: to construct, elaborate or expand.

In English: begin to build an opinion or idea.

Discuss

Talk or write about a topic, taking into account different issues and ideas.

Distinguish

Recognise point/s of difference.

Evaluate

Provide a detailed examination and substantiated judgement concerning the merit, significance or value of something.

In mathematics: calculate the value of a function at a particular value of its independent variables.

Explain

Provide additional information that demonstrates understanding of reasoning and/or application.

Familiar

Previously encountered in prior learning activities.

Identify

Establish or indicate who or what someone or something is.

Integrate

Combine elements.

Investigate

Plan, collect and interpret data/information and draw conclusions about.

Justify

Show how an argument or conclusion is right or reasonable.

Locate

Identify where something is found.

Manipulate

Adapt or change.

Non-routine

Non-routine problems: Problems solved using procedures not previously encountered in prior learning activities.

Reasonableness

Reasonableness of conclusions or judgements: the extent to which a conclusion or judgement is sound and makes sense

Reasoned

Reasoned argument/conclusion: one that is sound, well-grounded, considered and thought out.

Recognise

Be aware of or acknowledge.

Relate

Tell or report about happenings, events or circumstances.

Represent

Use words, images, symbols or signs to convey meaning.

Reproduce

Copy or make close imitation.

Responding

In English: When students listen to, read or view texts they interact with those texts to make meaning. Responding involves students identifying, selecting, describing, comprehending, imagining, interpreting, analysing and evaluating.

Routine problems

Routine problems: Problems solved using procedures encountered in prior learning activities.

Select

Choose in preference to another or others.

Sequence

Arrange in order.

Solve

Work out a correct solution to a problem.

Structured

Arranged in a given organised sequence.

In Mathematics: When students provide a structured solution, the solution follows an organised sequence provided by a third party.

Substantiate

Establish proof using evidence.

Succinct

Written briefly and clearly expressed.

Sustained

Consistency maintained throughout.

Synthesise

Combine elements (information/ideas/components) into a coherent whole.

Understand

Perceive what is meant, grasp an idea, and to be thoroughly familiar with.

Unfamiliar

Not previously encountered in prior learning activities.